MATLAB PROJECT 3

Please include this page in your Group file, as a front page. Type in the group number and the names of all members WHO PARTICIPATED in this project.

GROUP # 3

FIRST & LAST NAMES (UFID numbers are NOT required):

1. Jorge Castillo

2. Edrian Tirene

3. Pablo Garces

4. Anthony Ambiroso

5. Alihan

**By signing your names above, each of you had confirmed that you did the work and agree with the work submitted**.

Exercise 1

format compact

% printing subspace function

type subspace

function [] = subspace(A, B)

m = size(A, 1);

n = size(B, 1);

if m == n

sprintf('Col A and Col B are subspaces of R^ %i \n)', m);

k = rank(A);

p = rank(B);

fprintf('dim of Col A is k = %i\n',k);

fprintf('dim of Col B is p = %i\n',p);

if k == p

if k == rank([A B])

sprintf('Col A = Col B');

else

sprintf('k = p, the dimensions of Col A and Col B are the same, but Col A ~= Col B');

end

else

sprintf('k ~= p, the dimensions of Col A and Col B are different');

end

if (k == m) && (p == m)

fprintf('k = m (%i = %i) and p = m (%i = %i) Col A and Col B are all R^%i \n', k, m, p, m, m);

elseif k == m

fprintf('k = m (%i = %i) Col A is all R^%i \n', k, m, m);

elseif p == m

fprintf('p = m (%i = %i) Col B is all R^%i \n', p, m, m);

else

fprintf('Neither Col A or Col B are all in R^%i', m);

end

return

else

fprintf('Col A and Col B are subspaces of different spaces');

return

end

% part a

A = [2,-4,-2,3;6,-9,-5,8;2,-7,-3,9;4,-2,-2,-1;-6,3,3,4]

A =

2 -4 -2 3

6 -9 -5 8

2 -7 -3 9

4 -2 -2 -1

-6 3 3 4

B = rref(A)

B =

1.0000 0 -0.3333 0

0 1.0000 0.3333 0

0 0 0 1.0000

0 0 0 0

0 0 0 0

subspace(A,B)

dim of Col A is k = 3

dim of Col B is p = 3

Neither Col A or Col B are all in R^5

% part b

A = magic(4)

A =

16 2 3 13

5 11 10 8

9 7 6 12

4 14 15 1

B = eye(4)

B =

1 0 0 0

0 1 0 0

0 0 1 0

0 0 0 1

subspace(A,B)

dim of Col A is k = 3

dim of Col B is p = 4

p = m (4 = 4) Col B is all R^4

% part c

A = magic(4)

A =

16 2 3 13

5 11 10 8

9 7 6 12

4 14 15 1

B = eye(3)

B =

1 0 0

0 1 0

0 0 1

subspace(A,B)

Col A and Col B are subspaces of different spaces

% part d

A = magic(5)

A =

17 24 1 8 15

23 5 7 14 16

4 6 13 20 22

10 12 19 21 3

11 18 25 2 9

B = eye(5)

B =

1 0 0 0 0

0 1 0 0 0

0 0 1 0 0

0 0 0 1 0

0 0 0 0 1

subspace(A,B)

dim of Col A is k = 5

dim of Col B is p = 5

k = m (5 = 5) and p = m (5 = 5) Col A and Col B are all R^5

% Elementary row operations in general change the column space of a matrix. When A in part a. is row reduced, the last two rows are all zeroes so the columns do not span R^5.

diary save

Exercise 2

format compact

type shrink

function B=shrink(A)

    [~,pivot]=rref(A);

    B=A(:,pivot);

end

A=magic(4)

A =

    16     2     3    13

     5    11    10     8

     9     7     6    12

     4    14    15     1

%determines the pivot columns

[~,pivot]=rref(A)

pivot =

     1     2     3

%Above command gives us pivot columns

B=A(:,pivot)

B =

    16     2     3

     5    11    10

     9     7     6

     4    14    15

%Gives us the pivot columns

type basis

function B=basis(A)

    m=size(A,1);

    A=shrink(A);

    sprintf('a basis for Col A is \n')

    B=A

    if rank(B) == m

        sprintf('a basis for R^% i is \n', m)

    elseif rank(B) < m

        I = eye(m);

        D = horzcat(B,I);

        D = shrink(D);

        if rank(D) == m

            sprintf('a basis of R^% i is \n', m)

            B=D;

        end

    else

        disp ('What? It is not a basis!?')

    end

end

A = [1,0;0,0;0,0;0,1]

A =

     1     0

     0     0

     0     0

     0     1

B=basis(A)

ans =

    'a basis for Col A is

     '

B =

     1     0

     0     0

     0     0

     0     1

ans =

    'a basis of R^ 4 is

     '

B =

     1     0     0     0

     0     0     1     0

     0     0     0     1

     0     1     0     0

A=[2,0;4,0;1,0;0,0]

A =

     2     0

     4     0

     1     0

     0     0

B=basis(A)

ans =

    'a basis for Col A is

     '

B =

     2

     4

     1

     0

ans =

    'a basis of R^ 4 is

     '

B =

     2     1     0     0

     4     0     1     0

     1     0     0     0

     0     0     0     1

A=magic(3)

A =

     8     1     6

     3     5     7

     4     9     2

B=basis(A)

ans =

    'a basis for Col A is

     '

B =

     8     1     6

     3     5     7

     4     9     2

ans =

    'a basis for R^ 3 is

     '

B =

     8     1     6

     3     5     7

     4     9     2

A=magic(6)

A =

    35     1     6    26    19    24

     3    32     7    21    23    25

    31     9     2    22    27    20

     8    28    33    17    10    15

    30     5    34    12    14    16

     4    36    29    13    18    11

B=basis(A)

ans =

    'a basis for Col A is

     '

B =

    35     1     6    26    19

     3    32     7    21    23

    31     9     2    22    27

     8    28    33    17    10

    30     5    34    12    14

     4    36    29    13    18

ans =

    'a basis of R^ 6 is

B =

    35     1     6    26    19     1

     3    32     7    21    23     0

    31     9     2    22    27     0

     8    28    33    17    10     0

    30     5    34    12    14     0

     4    36    29    13    18     0

diary off

Exercise 3

diary on

type closetozeroroundoff

function B=closetozeroroundoff(A)

[m,n]=size(A);

for i=1:m

for j=1:n

if abs(A(i,j))<10^(-7)

A(i,j) = 0;

end

end

end

B = A;

end

type polyspace

function P = polyspace(B, Q, r)

format rat

% Take coeff of the polynomial B(1) -> 1 x n row vector

u = sym2poly(B(1));

n = length(u);

C = zeros(n);

for i = 1:n

C(:,i) = transpose( sym2poly(B(i)));

end

P = closetozeroroundoff(C);

if( rank(P) == n ) % forms a basis

% 1)

sprintf('The polynomials in B form a basis for for P%d',n-1)

% polynomial -> column vector

isomorphicColumnOfQ = transpose( sym2poly(Q) );

% solve linear system

reducedAugmented = rref([ P isomorphicColumnOfQ ]);

% column vector -> row vector

y = transpose(reducedAugmented( :, n + 1 ));

fprintf('the coordinates of the polynomial Q with respect to the basis P are%\n')

y = closetozeroroundoff(y)

% 2)

% B coord -> E coord ( standard basis )

q = P\*r;

q = transpose(closetozeroroundoff(q));

fprintf('the polynomial R with respect to the standard basis is%\n');

R = poly2sym(q)

else % does not form a basis

sprintf('the polynomials in B do not form a basis for P%d', n-1)

fprintf('the reduced echelon form of P is%\n')

A = rref(P)

return

end

end

syms x

% part a:

B=[x^3+3\*x^2,10^(-8)\*x^3+x,10^(-8)\*x^3+4\*x^2+x,x^3+x] ,

B =

[ x^3 + 3\*x^2, x^3/100000000 + x, x^3/100000000 + 4\*x^2 + x, x^3 + x]

Q=10^(-8)\*x^3+x^2+6\*x, r=[2;-3;1;0]

Q =

x^3/100000000 + x^2 + 6\*x

r =

2

-3

1

0

P = polyspace(B,Q,r)

ans =

'the polynomials in B do not form a basis for P3'

the reduced echelon form of P is

A =

1 0 0 1

0 1 0 7/4

0 0 1 -3/4

0 0 0 0

P =

1 0 0 1

3 0 4 0

0 1 1 1

0 0 0 0

% In part a, the reduced echelon form has a rank of 3 and is not in R4,

% this means it does not form a basis for P3

B=[x^3-1,10^(-8)\*x^3+2\*x^2,10^(-8)\*x^3+x,x^3+x]

B =

[ x^3 - 1, x^3/100000000 + 2\*x^2, x^3/100000000 + x, x^3 + x]

P = polyspace(B,Q,r)

ans =

'The polynomials in B form a basis for for P3'

the coordinates of the polynomial Q with respect to the basis P are

y =

0 1/2 6 0

the polynomial R with respect to the standard basis is

R =

2\*x^3 - 6\*x^2 + x - 2

P =

1 0 0 1

0 2 0 0

0 0 1 1

-1 0 0 0

B=[x^4+x^3+x^2+1,10^(-8)\*x^4+x^3+x^2+x+1,10^(-8)\*x^4+x^2+x+1, 10^(-8)\*x^4+x+1,10^(-8)\*x^4+1],

B =

[ x^4 + x^3 + x^2 + 1, x^4/100000000 + x^3 + x^2 + x + 1, x^4/100000000 + x^2 + x + 1, x^4/100000000 + x + 1, x^4/100000000 + 1]

Q=x^4-1, r=diag(magic(5))

Q =

x^4 - 1

r =

17

5

13

21

9

P = polyspace(B,Q,r)

ans =

'The polynomials in B form a basis for for P4'

the coordinates of the polynomial Q with respect to the basis P are

y =

1 -1 0 1 -2

the polynomial R with respect to the standard basis is

R =

17\*x^4 + 22\*x^3 + 35\*x^2 + 39\*x + 65

P =

1 0 0 0 0

1 1 0 0 0

1 1 1 0 0

0 1 1 1 0

1 1 1 1 1

diary off

Exercise 4

>> diary on

>> function [T, I] = reimsum(P,a,b,n)

N=length(n);

polynomial = sym2poly(P);

x\_mid = (a + b) / 2;

for j=1:N

% width of subinterval

height = (a-b)/(n(1,j));

% area = h \* p( x )

c(j) = height \* polyval( polynomial, b );

d(j) = height \* polyval( polynomial, x\_mid );

f(j) = height \* polyval( polynomial, a );

end

A = [ n; c; d; f ]';

T = array2table(A, 'VariableNames', {'n','Left','Middle','Right'});

I = double(int(P, a, b));

end

>> a=-1;b=1;n=[1:10];

>> syms x

>> P = 2\*x^4 + 4\*x^2 – 1;

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_ \_\_\_\_\_\_\_ \_\_\_\_\_\_\_ \_\_\_\_\_\_\_

1 -10 2 -10

2 -5 1 -5

3 -3.3333 0.66667 -3.3333

4 -2.5 0.5 -2.5

5 -2 0.4 -2

6 -1.6667 0.33333 -1.6667

7 -1.4286 0.28571 -1.4286

8 -1.25 0.25 -1.25

9 -1.1111 0.22222 -1.1111

10 -1 0.2 -1

I =

1.4667

>> n=[1,5,10,100,1000,10000];

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_

1 -10 2 -10

5 -2 0.4 -2

10 -1 0.2 -1

100 -0.1 0.02 -0.1

1000 -0.01 0.002 -0.01

10000 -0.001 0.0002 -0.001

I =

1.4667

>> n=[1:10];a=-10;b=10;

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_ \_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_

1 -4.0798e+05 20 -4.0798e+05

2 -2.0399e+05 10 -2.0399e+05

3 -1.3599e+05 6.6667 -1.3599e+05

4 -1.02e+05 5 -1.02e+05

5 -81596 4 -81596

6 -67997 3.3333 -67997

7 -58283 2.8571 -58283

8 -50998 2.5 -50998

9 -45331 2.2222 -45331

10 -40798 2 -40798

I =

8.2647e+04

>> n=[1,5,10,100,1000,10000];

>>

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_

1 -4.0798e+05 20 -4.0798e+05

5 -81596 4 -81596

10 -40798 2 -40798

100 -4079.8 0.2 -4079.8

1000 -407.98 0.02 -407.98

10000 -40.798 0.002 -40.798

I =

8.2647e+04

>> P=x^3-2\*x

P =

x^3 - 2\*x

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_

1 -19600 0 19600

5 -3920 0 3920

10 -1960 0 1960

100 -196 0 196

1000 -19.6 0 19.6

10000 -1.96 0 1.96

I =

0

>> n=[1:10];

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_ \_\_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_

1 -19600 0 19600

2 -9800 0 9800

3 -6533.3 0 6533.3

4 -4900 0 4900

5 -3920 0 3920

6 -3266.7 0 3266.7

7 -2800 0 2800

8 -2450 0 2450

9 -2177.8 0 2177.8

10 -1960 0 1960

I =

0

>> n=[1,5,10,100,1000,10000];a=-1;b=1;

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_\_

1 2 0 -2

5 0.4 0 -0.4

10 0.2 0 -0.2

100 0.02 0 -0.02

1000 0.002 0 -0.002

10000 0.0002 0 -0.0002

I =

0

>> n=[1:10];

>> [T,I]=reimsum(P,a,b,n)

T =

n Left Middle Right

\_\_ \_\_\_\_\_\_\_ \_\_\_\_\_\_ \_\_\_\_\_\_\_\_

1 2 0 -2

2 1 0 -1

3 0.66667 0 -0.66667

4 0.5 0 -0.5

5 0.4 0 -0.4

6 0.33333 0 -0.33333

7 0.28571 0 -0.28571

8 0.25 0 -0.25

9 0.22222 0 -0.22222

10 0.2 0 -0.2

I =

0

Exercise 5

diary on

format compact

type polint

function B = polint( P )

syms x;

for n = 1:length(P) % integral

I(n) = (P(n))/(length(P) - n + 1);

end

I(length(P)+1) = 1; % add constant

B = poly2sym(I,x);

end

%a

P = [6,5,4,3,2,6]

P =

6 5 4 3 2 6

polint(P)

ans =

x^6 + x^5 + x^4 + x^3 + x^2 + 6\*x + 1

int(poly2sym(P,x)) + 1

ans =

x^6 + x^5 + x^4 + x^3 + x^2 + 6\*x + 1

%b

P = [1,0,-1,0,3,0,1]

P =

1 0 -1 0 3 0 1

polint(P)

ans =

x^7/7 - x^5/5 + x^3 + x + 1

int(poly2sym(P,x)) + 1

ans =

x^7/7 - x^5/5 + x^3 + x + 1

Exercise 6

type markov

function q = markov(P,x0)

k = 0;

xk = x0;

n = size(P,1);

s = (sum(P,1));

S = ones(1,n);

if s == S

Q = null(P-eye(n),'r');

c = sum(Q);

q = (1./c).\*Q;

while norm(xk-q) > 10^(-7)

xk = P\*xk;

k = k + 1;

end

k

xk

else

disp('P is not a stochastic matrix')

q = [];

end

end

P = [.6,.3;.5,.7];

x0 = [.4;.6];

q = markov(P,x0)

P is not a stochastic matrix

q =

[]

%b

P = [.5,.3;.5,.7];

q = markov(P,x0)

k =

8

xk =

0.3750

0.6250

q =

0.3750

0.6250

% xk and q are the same vector

%c

P = [.9,.2;.1,.8];

x0 = [.12;.88];

q = markov(P,x0)

k =

45

xk =

0.6667

0.3333

q =

0.6667

0.3333

%d

x0 = [.14;.86];

q = markov(P,x0)

k =

45

xk =

0.6667

0.3333

q =

0.6667

0.3333

x0 = [.86;.14];

q = markov(P,x0)

k =

42

xk =

0.6667

0.3333

q =

0.6667

0.3333

% q in part (d) and (c) is the same

%the output is not affected by different x0 vector elements or order as long as those element add up to one

%the elements can contribute to a change in the number of k (interation)

%e

P = [.9,.01,.09;.01,.9,.01;.09,.09,.9];

x0 = [.5;.3;.2];

q = markov(P,x0)

k =

128

xk =

0.4354

0.0909

0.4737

q =

0.4354

0.0909

0.4737

% Initia

diary off